

Writing Biconditional Statements // Truth Tables

Lesson Objective

INBAT write biconditional statements; ~~and~~
 use ~~then~~ biconditional statements to write conditional
 & converse; fill out truth tables.

Using Definitions

You can write a definition as a conditional statement in "if-then" form or as its converse. Both the conditional statement AND converse are true for definitions. For example, consider the following definition of perpendicular lines:

$\hookrightarrow$ If two lines intersect to form a right angle, then they are perpendicular lines.

Now, write the converse of the definition:

If 2 lines are perpendicular ($\perp$), then they intersect to form a right angle.

Example: Use the diagram. Decide whether each statement is true. Explain your answer using the definitions you have learned from Chapter 1 and above.

1. $\angle JMF$ and $\angle FMG$ are supplementary.

T

2. Point M is the midpoint of $\overline{FH}$.

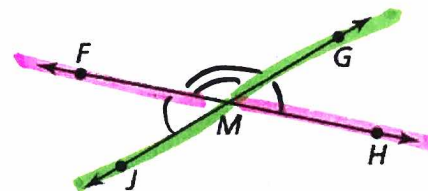
F

3. $\angle JMF$ and $\angle HMG$ are vertical angles.

T

4. $\overline{FH} \perp \overline{JG}$

F



Biconditional Statements: When a conditional statement and its converse are both TRUE, you can write them as a single biconditional statement.

$\rightarrow$ A biconditional statement is a statement that contains the phrase "if and only if"

Words: p if and only if q Symbols: $p \leftrightarrow q$

**We saw above that definitions have true conditional and converse statements. So, ANY definition can be written as biconditional statements!

Example:

5. Rewrite the definition of perpendicular lines above as a single biconditional statement.

2 lines intersect to form a right angle
 if and only if they are perpendicular lines.

6. Rewrite the definition of congruent segments (shown below) as a single biconditional statement:

"If two line segments have the same length, then they are congruent segments."

2 line segments have the same length if and only if they are congruent segments.

7. Write the conditional statement and converse within the biconditional.

"An angle is a right angle if and only if its measure is 90° ."

Conditional: If an $\angle$ is a right $\angle$, then its measure is 90° .

Converse:

If an $\angle$'s measure is 90° , then it is a right $\angle$.

Truth Tables

Truth Value: The truth value of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**.

Let's look at conditional statements bring TRUE or FALSE in the context of keeping a promise.

Conditional		
p	q	$p \rightarrow q$
1. T	T	T
2. T	F	F
3. F	T	T
4. F	F	T

MY PROMISE: If there is an elephant in the library, then I will give you \$100.

1. Suppose there really T IS an elephant in the library, and I T give you \$100.

2. Suppose there really T IS an elephant in the library, and I don't F give you any money.

3. Suppose there is not an elephant in the library, but I give you \$100 anyway. (SWEET!)

4. Suppose there is not an elephant in the library, and I don't give you any money.

Now, let's fill out the following truth tables for the converse, inverse, and contrapositive of a conditional statement:

Converse		
p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive				
p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Think About It... Two statements are logically equivalent when they have the same truth table.

So, which statements define the term above? (HINT: look at their truth table outcomes!)

conditional and contrapositive
converse and inverse